國立清華大學命題紙

九十一 學年度第二學期 電機工程 學系 博士班研究生資格考試

科目 電磁理論 科號 共 3 頁第 1 頁 ＊請在試卷(答案卷)內作答

1. Electromagnetic waves satisfy all of Maxwell's equations. Consider, in free space, the following complex electric field vectors:

   (i) $\mathbf{E}_1 = \mathbf{a}_z e^{jky}$

   (ii) $\mathbf{E}_2 = \mathbf{a}_z e^{-jkz}$

   (iii) $\mathbf{E}_3 = (\mathbf{a}_x + j\mathbf{a}_z)e^{-jky}$

   (iv) $\mathbf{E}_3 = (2j\mathbf{a}_x - \mathbf{a}_y)e^{jkz}$

   (v) $\mathbf{E}_3 = (\mathbf{a}_x + \mathbf{a}_z)e^{-jk(x-z)/\sqrt{2}}$

(a) Can these electric field vectors satisfy the Helmholtz equation

$(\nabla^2 + \omega^2 \mu_0 \varepsilon_0)\mathbf{E} = 0$? If so, determine the corresponding dispersion relation. (5%)

(b) Which of them represent electromagnetic waves? For those which do not, specify which of Maxwell's equations are violated. For those which do, (i) determine the corresponding complex magnetic field vector **H** and (ii) give the corresponding real time-domain representations for both **E** and **H**. (10%)

2. The magnetic field **H** and electric field **E** of a Hertzian dipole at very large distances ($kr >> 1$) are

$$\mathbf{E} = -\mathbf{a}_\theta \frac{k^2 q\ell}{4\pi\varepsilon_0 r}\sin\theta\cos(\omega t - kr)$$

$$\mathbf{H} = -\mathbf{a}_\phi \frac{\omega k q\ell}{4\pi r}\sin\theta\cos(\omega t - kr)$$

(a) Find the Poynting's power density vector **S** as a function of time. What is the time-averaged power density vector **S**? (5%)

(b) By integrating the Poynting vector over the surface of a sphere of radius r, find the time-averaged power P radiated by the Hertzian dipole. (5%)

(c) The amplitude of the current in the Hertzian dipole is $I_o = \omega q$. By using

$P = \frac{1}{2} I_0^2 R_{rad}$, find the radiation resistance R_{rad} of the Hertzian dipole. (5%)

(d) A radio station is 15 km away from a city. The transmitting antenna tower may be modeled as a Hertzian dipole antenna of dipole moment $q\ell$. To maintain the FCC standard of 25 mV/m field strength in the city, how much radiation power P must be provided? (5%)

3. We learned the method of separating variables when we tried to solve electrostatic problems.
(i) Please explain why this method will work with fields confined by a rectangular, cylindrical, or spherical object. (3%)
(ii) Can we use the same method to solve the fields confined by objects of arbitrary shapes? If not, how can such fields be solved? (2%)

4. Please derive the Poynting theorem. Please also name each term appeared in the equation. (10%)

5. The data rate of today's personal computers is very fast and these computers wired together. Can you randomly disconnect a computer from the network or without doing any arrangement?
(i) Please discuss what negative consequences may happen. Please also explain the physical result causes these problems. (4%)
(ii) Please discuss what you should do (but you still need this computer to do something elsewhere) if such an unwanted thing happens. (2%)

6. Any communication channel or medium has a certain bandwidth.
(i) Please give typical bandwidths of parallel transmission lines, coaxial cables and metallic rectangular waveguides. (3%)
(ii) Please explain why the physical phenomena limit the bandwidths of each of the above three waveguides. (6%)

7. For applications, we sometimes arrange antennas as arrays of a certain order instead of using a single antenna with an equivalent net power. Please explain why. Please also explain the working principle of arrayed antennas. (5%)

8. Green's function relates sources to the potentials and hence plays an important role in electromagnetics. For a source placed at the origin of spherical coordinates, it is known that Green's function G is given by

$$G(r) = \frac{e^{-jk_0 r}}{4\pi r},$$

where r is the radial distance in spherical coordinates and k_0 is the propagation constant in free space. Find $\nabla^2 G(r)$ at $r \neq 0$. (10%)

9. Consider a plane wave in free space with electric field is $\mathbf{E}(\mathbf{r}) = \hat{x}E_0 e^{-jk_0 z}$, where E_0 is a constant. Find the associated magnetic vector potential $\mathbf{A}(\mathbf{r})$. You can made a self-consistent assumption on the divergence of $\mathbf{A}$. (10%)

10. The Lorentz force law describes the electromagnetic force exerted on a particle of charge q and velocity $\mathbf{v}$. This force can be expressed in several ways. Write down the Lorentz force law in terms of
   (a) electric field $\mathbf{E}(\mathbf{r}, t)$ and magnetic field $\mathbf{B}(\mathbf{r}, t)$, (5%)
   (b) electric scalar potential $\Phi(\mathbf{r}, t)$ and magnetic vector potential $\mathbf{A}(\mathbf{r}, t)$. (5%)

Constants and Formulas for Reference

$\epsilon_0 \simeq 8.854 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ H/m

In spherical coordinates, $\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right)$