國 立 清 華 大 學 命 題 紙

九十八學年度第二學期 光電工程研究所 博士班研究生資格考試

科目 電磁理論 科號 共 頁第 頁 ＊請在試卷(答案卷)內作答

1. (10 points) Assuming each atom contributes one free electron, the density ρ of mobile charges in a piece of copper can be expressed as $\rho = \frac{eNd}{M}$, where e is the charge of electron, N is the Avogardro's number, M is atomic mass of copper, and d is density of copper. Given the density of copper is 9 gm/cm^3, ρ is estimated to be 1.4×10^4 C/cm^3 ($\mu_0 = 4\pi\times10^{-7}$ H/m).

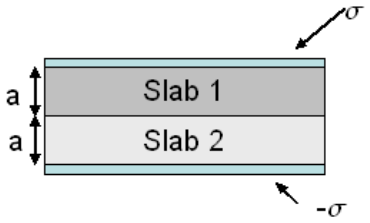
1a. (3%) Please calculate the average electron velocity in a copper wire (1 mm in diameter) carrying a current of 1A.

1b. (3%) What is the force of attraction per unit length between two such wires that are 1 cm apart?

1c. (4%) If the stationary positive ions are somehow removed, there would be a repulsion force between the wires due to electric force. How many times greater than the magnetic force is it?

2. (6 points) The plane boundary defined by z = 0 separates air from a block of iron. If $\vec{B}_1 = 4\hat{a}_x - 6\hat{a}_y + 8\hat{a}_z$ (Wb/m^2) in the air ($z \geq 0$), please find $\vec{B}_2$ in iron ($z \leq 0$), given that μ= $5000\mu_0$ for iron and $\vec{J}_s = 0$.

3. (9 points) The space between the plates of a parallel-plate capacitor is filled with two slabs of linear dielectric material. Each slab has thickness a, so the total distance between the plates is 2a. Slab 1 has a dielectric constant of 2, and slab 2 has a dielectric constant of 1.5. The free charge density on the top plate is σ and on the bottom plate -σ. (Please show your calculation for each question.)



3a. (3 %) Please find the magnitude ratio for the electric field in slab 1 and slab 2 ($\frac{|\vec{E}_1|}{|\vec{E}_2|}$).

3b. (3 %) Please find the magnitude ratio for the polarization in slab 1 and slab 2 ($\frac{|\vec{P}_1|}{|\vec{P}_2|}$).

3c. (3 %) Please find the potential difference between the plates.

4. (10 points) You have been learning about plane-waves for a long time, give three daily examples of plane-waves. Be specific on your descriptions.

5. (20 points) You have *N* plane-waves propagating in vacuum in the z-direction with equal electric field amplitudes (for example, one). The angular frequency difference between any two plane-waves is $\Delta\omega$. Each plane-wave has it's own time-invariant phase.

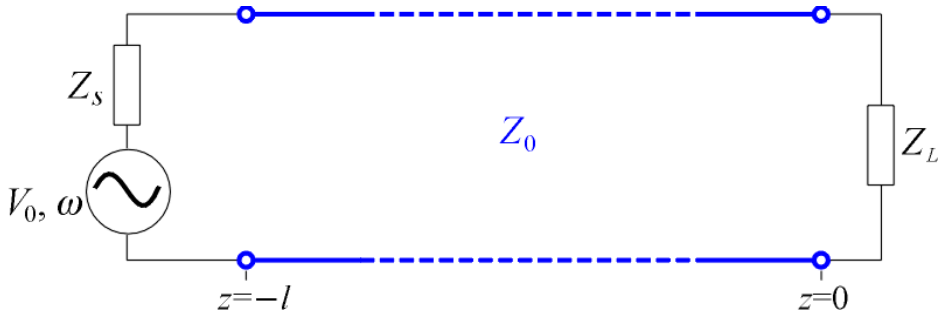
5a. (8%) Derive the general mathematical expression for the total electric fields.

5b. (6%) Plot the power at z=0 for random phases. Be specific on all scales.

5c. (6%) Plot the power at z=0 for equal phases. Be specific on all scales.

6. (25 points) Consider a transmission line as shown below, where the characteristic impedance is $Z_0 = 50\ \Omega$, the phase velocity of the voltage/current waves is $v_p = 2\times10^8$ m/s, the length of the line is $l =$ 3.2 m, the load impedance is $Z_L = (50 - j100)\Omega$, the sinusoidal voltage source has an

amplitude of $V_0 = 100$ V, an angular frequency of $\omega = 2\pi \times 125\,\text{MHz}$, and an internal impedance of $Z_S = 50\,\Omega$.



6a. (5%) What is the meaning of the characteristic impedance Z_0?

6b. (5%) The line impedance is formulated as:

$$Z(z) = Z_0 \frac{Z_L - jZ_0 \tan(\beta z)}{Z_0 - jZ_L \tan(\beta z)}.$$

What is the equivalent impedance Z_{in} of the "loaded transmission line" seen by an observer at the source end ($z = -l$) looking toward to the right hand side?

6c. (5%) What is the magnitude and phase of the current phasor I_S flowing through the load Z_L?

6d. (5%) What are the powers supplied by the source P_{tot}, and delivered to the load P_L, respectively?

6e. (5%) How do you maximize the power delivered to the load P_L?

7. (20 points)

7a. (10%) For TE and TM modes of the same order in the same symmetric dielectric planar waveguide, which one has a larger propagation constant? Which one has a higher cutoff frequency?

7b. (10%) Why dielectric waveguides don't support TEM modes?