國 立 清 華 大 學 命 題 紙

一百零三學年度第二學期 光電工程研究所 博士班研究生資格考試

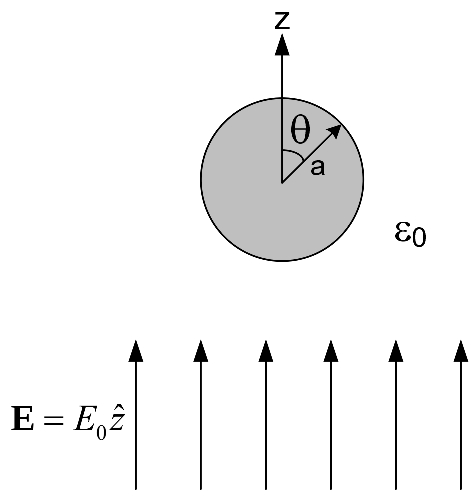
科目 電磁理論 共____頁第____頁 ＊請在試卷(答案卷)內作答

1. (15%) In the absence of time variation, the four fundamental postulates of electrostatics and magnetostatics are $\{\nabla\times\vec{E}=0,\ \nabla\cdot\vec{D}=\rho,\ \nabla\times\vec{H}=\vec{J},\ \nabla\cdot\vec{B}=0\}$.

   (a) (5%) In the presence of time variation, the first postulate is modified as $\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$. What is the physical law from which this modification is made? (Hint: Take surface integral for both sides of equality.)

   (b) (10%) In the presence of time variation, the third postulate is modified as $\nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}$. What are the assumptions of medium such that the four first-order differential equations can be combined into a single second-order wave equation for the electric field $\nabla^2\vec{E}-\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}=0$. (Hint: No need to deduce the wave equation. For example, the medium has to be "linear".)

2. (15%) An ideal metallic sphere of radius a is positioned in free-space with an uniform applied electric field intensity of $\mathbf{E}=E_0\hat{z}$. Discuss and provide a detailed sketch showing the electric field lines of the sphere and its vicinity. Also show the distribution of the surface charges.



3. (10%) Please derive the general expression of cut-off wavelength for TEn mode of a parallel-plate metallic waveguide with separation b between the plates and then derive the general formula of cut-off frequency in terms of b, ε, μ, and mode number n.

4. (15%) When there is a relative motion between a time-harmonic source and a receiver, the frequency of the wave detected by the receiver tends to be different from that emitted by the source. This phenomenon is known as the Doppler effect. Let us assume that a light transmitter of a time-harmonic wave of a frequency f moves with a velocity u (assume u << c) at an angle θ relative to the direct line to a stationary receiver.

(a) (5%) Please derive and show that the frequency of the received wave is $f' = \dfrac{f}{1 - \dfrac{u}{c}\cos\theta}$.

(b) (5%) If the transmitted signal has a spectral linewidth of $\Delta\upsilon$, what would be the linewidth of the received signal after the Doppler effect?

(c) (5%) If the stationary receiver (target) has a rough surface comparable to the wavelength of the light, how would the linewidth of the received signal change? Why?

5. (4%) What is the major consequence of $\nabla \cdot \vec{B} = 0$ to the magnetic field lines, as compared with the consequence of $\nabla \cdot \vec{D} = \rho$ to the electric field lines, where B is the magnetic flux density, D is the electric flux density, and ρ is a volume charge density? To receive credits, you have to explain and justify your answers. You are encouraged to provide drawings that illustrate the concepts.

6. (6%) A very long solenoid contains an iron core with a cross sectional area of S and has n turns per unit length carrying a current I. The permeability of the iron core is μ. (1) What is the magnetic energy per unit length stored in the solenoid? (2) What is the inductance per unit length of this solenoid?

7. (7%) What is the ratio of the wavelengths of a sinusoidal plane wave of frequency f if it propagates in perfect dielectric of permittivities ε_1 and ε_2, and permeability μ_0?

(a) $\frac{\lambda_1}{\lambda_2} = \sqrt{\varepsilon_1 / \varepsilon_2}$ (b) $\frac{\lambda_1}{\lambda_2} = \sqrt{\varepsilon_2 / \varepsilon_1}$ (c) $\frac{\lambda_1}{\lambda_2} = \varepsilon_2 / \varepsilon_1$.

8. (8%) By measurements it was found that the time-average power of the sun's radiation on the surface of the earth is about 1.35 kW/m^2, for normal incident of the plane waves from the sun. This radiation is composed of a very wide band of frequencies, and the components of different frequencies are generally polarized elliptically. Assuming, for simplicity, that the entire radiation is a linearly polarized wave of a single frequency, determine the rms value its electric and magnetic field. [Hint $\mu_0 = 4\pi\times10^{-7}$ V·s/(A·m). $\varepsilon_0 = 8.8541878176.. \times 10^{-12}$ F/m]

(a) E=713 V/m, H=1.9 A/m (b) E=356 V/m, H=0.95 A/m (c) E=178 V/m, H=0.48 A/m.

9. (10%) Please plot an equivalent electrical circuit for a lossy transmission line (non-perfect dielectric and non-perfect conducting material).

10. (10%) Consider a rectangular waveguide, infinitely long in the z-direction, with a width (x-direction) *2* cm and a height (y-direction) *1* cm. With the wave equation which describes the **E** and **B** fields of the lowest modes, and find the phase velocity and group velocity, accordingly, if the lowest mode can propagate.